MODELS OF TURBULENT MOMENTUM AND HEAT TRANSFER IN A DISPERSED PHASE BASED ON EQUATIONS FOR THE SECOND AND THIRD MOMENTS OF PARTICLE VELOCITY AND TEMPERATURE PULSATIONS

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Models of various complexity for describing the hydrodynamics and heat transfer of particles in turbulent flows are proposed on the basis of the chain of equations for the moments of velocity and temperature, obtained from an equation for the probability density function.

In recent years along with local-equilibrium algebraic models for description of turbulent momentum and heat transfer in the dispersed phase of a two-phase flow [1-10], ever increasing use is made of differential models based on equations for the balance of turbulent energy or the second moments of pulsations of particle velocity and temperature [11-14]. The employment of differential models makes it possible to describe nonlocal effects of transfer of velocity and temperature pulsations by inertial particles — the convective and diffusive mechanisms of turbulent momentum and heat transfer. A progressive method of constructing the system of equations for the description of dynamics and heat exchange in the dispersed phase of a two-phase flow is the employment of a kinetic equation for the probability density function (PDF) of the particle velocity and temperature in a turbulent flow [15, 16].

The present work refines the equation obtained in [15, 16] for PDF, presents the chain of equations for the moments of velocity and temperature, and proposes various schemes of closing the system of equations for the dispersed phase at the level of equations for the first, second or third moments. The analysis is performed under the assumption that the characteristics of the turbulent carrying flow are known, while the volume concentration is insignificant, and collisions between the particles can be ignored. The generation of pulsations of the dispersed phase characteristics in this case is caused by the particle interaction with turbulent pulsations of the carrying flow which are modeled by Gaussian random functions.

1. Within the framework of the assumption that the density of the dispersed phase material is substantially larger than that of the carrying gas flow, equations of motion and heat transfer for a single particle are written in the form

$$\frac{dR_{pi}}{d\tau} = v_{pi}, \quad \frac{dv_{pi}}{d\tau} = \frac{u_i(\mathbf{R}_p(\tau), \tau) - v_{pi}}{\tau_u} + F_i(\mathbf{R}_p(\tau), \tau), \tag{1}$$

$$\frac{d\vartheta_p}{d\tau} = \frac{t(\mathbf{R}_p(\tau), \tau) - \vartheta_p}{\tau_t} + Q(\mathbf{R}_p(\tau), \tau).$$

Expressions (1) represent Langevin-type equations, since velocity u and temperature t of the carrying turbulent flow, entering into them, are random functions. To go from dynamic stochastic equations (1) to the statistical description of the particle distribution in velocity and temperature we introduce the PDF

$$P(\mathbf{x}, \mathbf{v}, \vartheta, \tau) = \langle p \rangle = \langle \delta(\mathbf{x} - \mathbf{R}_p(\tau))\, \delta(\mathbf{v} - \mathbf{v}_p(\tau))\, \delta(\vartheta - \vartheta_p(\tau)) \rangle, \tag{2}$$

where averaging is performed over the realization ensemble of random velocity and temperature fields.

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Differentiating (2) with respect to time, in view of (1), we obtain the following equation for PDF:

$$\frac{\partial P}{\partial \tau}+v_k\frac{\partial P}{\partial x_k}+\frac{\partial}{\partial v_k}\left(\frac{U_k-v_k}{\tau_u}+F_k\right)P+\frac{\partial}{\partial \vartheta}\left(\frac{T-\vartheta}{\tau_t}+\right.$$
$$\left.+Q\right)P=-\frac{1}{\tau_u}\frac{\partial\langle u_k' p\rangle}{\partial v_k}-\frac{1}{\tau_t}\frac{\partial\langle\ 'p\rangle}{\partial\vartheta}\ . \tag{3}$$

The correlators $\langle u_k' p\rangle$ and $\langle t'p\rangle$ in Eq. (3) are determined using E. A. Novikov's relation [17] for Gaussian random functions:

$$\langle u_i' p\rangle=\iint\langle u_i'(\mathbf{x},\ \tau)\ u_k'(\mathbf{x}_1,\ \tau_1)\rangle\left\langle\frac{\delta p(\mathbf{x},\ \mathbf{v},\ \vartheta,\ \tau)}{\delta u_k(\mathbf{x}_1,\ \tau_1)}\right\rangle d\mathbf{x}_1 d\tau_1+$$
$$+\iint\langle u_i'(\mathbf{x},\ \tau)\,t'(\mathbf{x}_1,\ \tau_1)\rangle\left\langle\frac{\delta p(\mathbf{x},\ \mathbf{v},\ \vartheta,\ \tau)}{\delta t(\mathbf{x}_1,\ \tau_1)}\right\rangle dx_1 d\tau_1,$$
$$\langle t'p\rangle=\iint\langle t'(\mathbf{x},\ \tau)\,t'(\mathbf{x}_1,\ \tau)\rangle\left\langle\frac{dp(\mathbf{x},\ \mathbf{v},\ \vartheta,\ \tau)}{\delta t(\mathbf{x}_1,\ \tau_1)}\right\rangle d\mathbf{x}_1 d\tau_1+ \tag{4}$$
$$+\iint\langle u_k'(\mathbf{x}_1,\ \tau_1)\,t'(\mathbf{x},\tau)\rangle\left\langle\frac{\delta p(\mathbf{x},\ \mathbf{v},\ \vartheta,\ \tau)}{\delta u_k(\mathbf{x}_1,\ \tau_1)}\right\rangle d\mathbf{x}_1 d\tau_1,$$

where

$$\left\langle\frac{\delta p(\mathbf{x},\ \mathbf{v},\ \vartheta,\ \tau)}{\delta u_k(\mathbf{x}_1,\ \tau)}\right\rangle=-\frac{\partial}{\partial x_j}\left\langle p(\mathbf{x},\ \mathbf{v},\ \vartheta,\ \tau)\frac{\delta R_{pj}(\tau)}{\delta u_k(\mathbf{x}_1,\ \tau_1)}\right\rangle-$$
$$-\frac{\partial}{\partial v_j}\left\langle p(\mathbf{x},\ \mathbf{v},\ \vartheta,\ \tau)\frac{\delta v_{pj}(\tau)}{\delta u_k(\mathbf{x}_1,\ \tau_1)}\right\rangle-\frac{\partial}{\partial\vartheta}\left\langle p(\mathbf{x},\ \mathbf{v},\ \vartheta,\ \tau)\times\right.$$
$$\left.\times\frac{\delta\vartheta_p(\tau)}{\delta u_k(\mathbf{x}_1,\ \tau_1)}\right\rangle,$$
$$\left\langle\frac{\delta p(\mathbf{x},\ \mathbf{v},\ \vartheta,\ \tau)}{\delta t(\mathbf{x}_1,\ \tau)}\right\rangle=-\frac{\partial}{\partial\vartheta}\left\langle p(\mathbf{x},\ \mathbf{v},\ \vartheta,\ \tau)\frac{\delta\vartheta_p(\tau)}{\delta t(\mathbf{x}_1,\ \tau_1)}\right\rangle.$$

To find functional derivatives in (4), use is made of the solutions for the equations of motion and heat transfer of a single particle (1):

$$R_{pi}(\tau)=\int_0^\tau v_{pi}(\tau_1)\,d\tau_1,$$
$$v_{pi}(\tau)=\int_0^\tau\left[\frac{u_i(\mathbf{R}_p(\tau_1),\ \tau_1)}{\tau_u}+F_i(\mathbf{R}_p(\tau_1),\ \tau_1)\right]\exp\left(-\frac{\tau-\tau_1}{\tau_u}\right)d\tau_1, \tag{5}$$
$$\vartheta_p(\tau)=\int_0^\tau\left[\frac{t(\mathbf{R}_p(\tau_1),\ \tau_1)}{\tau_t}+Q(\mathbf{R}_p(\tau_1),\ \tau_1)\right]\exp\left(-\frac{\tau-\tau_1}{\tau_t}\right)d\tau_1.$$

By applying a functional differentiation operator to (5) we obtain a system of integral equations for determining the functional derivatives:

$$\frac{\delta R_{pi}(\tau)}{\delta u_j(\mathbf{x}_1,\ \tau_1)}=\delta_{ij}\left[1-\exp\left(-\frac{\tau-\tau_1}{\tau_u}\right)\right]\delta(\mathbf{x}_1-\mathbf{R}_p(\tau_1))\,\eta(\tau-\tau_1)+$$
$$+\int_{\tau_1}^{\tau}\left[1-\exp\left(-\frac{\tau-\tau_2}{\tau_u}\right)\right]\frac{\partial}{\partial x_n}\left[u_i(\mathbf{R}_p(\tau_2),\ \tau_2)+\tau_u F_i(\mathbf{R}_p(\tau_2),\ \tau_2)\right]\times \tag{6}$$

$$\times \frac{\delta R_{pn}(\tau_2)}{\delta u_j(\mathbf{x}_1, \tau_1)} d\tau_2, \tag{7}$$

$$\frac{\delta v_{pi}(\tau)}{\delta u_j(\mathbf{x}_1, \tau_1)} = \frac{\delta_{ij}}{\tau_u} \exp\left(-\frac{\tau-\tau_1}{\tau_u}\right) \delta(\mathbf{x}_1 - \mathbf{R}_p(\tau_1)) \eta(\tau-\tau_1) +$$

$$+ \frac{1}{\tau_u} \int_{\tau_1}^{\tau} \exp\left(-\frac{\tau-\tau_2}{\tau_u}\right) \frac{\partial}{\partial x_n} [u_i(\mathbf{R}_p(\tau_2), \tau_2) + \tau_u F_i(\mathbf{R}_p(\tau_2), \tau_2)] \times$$

$$\times \frac{\delta R_{pn}(\tau_2)}{\delta u_j(\mathbf{x}_1, \tau_1)} d\tau_2, \tag{8}$$

$$\frac{\delta \vartheta_p(\tau)}{\delta u_j(\mathbf{x}_1, \tau_1)} = \frac{1}{\tau_t} \int_{\tau_1}^{\tau} \exp\left(-\frac{\tau-\tau_2}{\tau_t}\right) \times$$

$$\times \frac{\partial}{\partial x_n} [t(\mathbf{R}_p(\tau_2), \tau_2) + \tau_t Q(\mathbf{R}_p(\tau_2), \tau_2)] \frac{\delta R_{pn}(\tau_2)}{\delta u_j(\mathbf{x}_1, \tau_1)} d\tau_2, \tag{9}$$

$$\frac{\delta \vartheta_p(\tau)}{\delta t(\mathbf{x}_1, \tau_1)} = \frac{1}{\tau_t} \exp\left(-\frac{\tau-\tau_1}{\tau_t}\right) \delta(\mathbf{x}_1 - \mathbf{R}_p(\tau_1)) \eta(\tau-\tau_1).$$

With the aim of obtaining closed expressions for the functional derivatives and correspondingly for the correlators $\langle u'_i p\rangle$ and $\langle t'p\rangle$ in [16] all the integral terms in (6)-(8) were excluded from consideration. However, the influence of these terms in nonuniform flows may be substantial, so in contrast to [16] we will disregard only the integral term in (6) and will approximately represent the integral terms in (7) and (8), in view of (5), in the form

$$\begin{aligned}
\frac{1}{\tau_u} \int_{\tau_1}^{\tau} \ldots &= \frac{\partial v_{pi}(\mathbf{R}_p(\tau), \tau)}{\partial x_n} \frac{\delta R_{pn}(\tau)}{\delta u_j(\mathbf{x}_1, \tau_1)}, \\
\frac{1}{\tau_t} \int_{\tau_1}^{\tau} \ldots &= \frac{\partial \vartheta_p(\mathbf{R}_p(\tau), \tau)}{\partial x_n} \frac{\delta R_{pn}(\tau)}{\delta u_j(\mathbf{x}_1, \tau_1)}.
\end{aligned} \tag{10}$$

In view of (6)-(10), expressions (4) will take the form:

$$\begin{aligned}
\langle u'_i p \rangle &= -\tau_u g_u \langle u'_i u'_k \rangle \left(\frac{\partial P}{\partial x_k} + \frac{\partial V_n}{\partial x_k} \frac{\partial P}{\partial v_n} + \frac{\partial \Theta}{\partial x_k} \frac{\partial P}{\partial \vartheta}\right) - \\
&\quad - f_u \langle u'_i u'_k \rangle \frac{\partial P}{\partial v_k} - f_{tu} \langle u'_i t' \rangle \frac{\partial P}{\partial \vartheta}, \\
\langle t'p \rangle &= -\tau_u g_{ut} \langle u'_k t' \rangle \left(\frac{\partial P}{\partial x_k} + \frac{\partial V_n}{\partial x_k} \frac{\partial P}{\partial v_n} + \frac{\partial \Theta}{\partial x_k} \frac{\partial P}{\partial \vartheta}\right) - \\
&\quad - f_{ut} \langle u'_k t' \rangle \frac{\partial P}{\partial v_k} - f_t \langle t'^2 \rangle \frac{\partial P}{\partial \vartheta},
\end{aligned} \tag{11}$$

where the coefficients f and g are found from the relations

$$g_u = \frac{1}{\tau_u \langle u'_i(\mathbf{x}, \tau) u'_k(\mathbf{x}, \tau) \rangle} \int_0^{\infty} \langle u'_i(\mathbf{x}, \tau) u'_k(\mathbf{R}_p(\tau_1), \tau_1) \rangle \left[1 - \exp\left(-\frac{\tau-\tau_1}{\tau_u}\right)\right] d\tau_1,$$

$$f_u = \frac{1}{\tau_u \langle u'_i(\mathbf{x}, \tau) u'_k(\mathbf{x}, \tau) \rangle} \int_0^{\infty} \langle u'_i(\mathbf{x}, \tau) u'_k(\mathbf{R}_p(\tau_1), \tau_1) \rangle \times \exp\left(-\frac{\tau-\tau_1}{\tau_u}\right) d\tau_1,$$

$$f_{tu}=\frac{1}{\tau_t\langle u_i'(\mathbf{x},\ \tau)t'(\mathbf{x},\ \tau)\rangle}\int\limits_0^\infty\langle u_i'(\mathbf{x},\ \tau)t'(\mathbf{R}_p(\tau_1),\ \tau_1)\rangle\times$$
$$\times\exp\left(-\frac{\tau-\tau_1}{\tau_t}\right)d\tau_1,$$

$$g_{ut}=\frac{1}{\tau_u\langle u_i'(\mathbf{x},\ \tau)t'(\mathbf{x},\ \tau)\rangle}\int\limits_0^\infty\langle u_i'(\mathbf{R}_p(\tau_1),\ \tau_1)t'(\mathbf{x},\ \tau)\rangle\Bigg[1-$$
$$-\exp\left(-\frac{\tau-\tau_1}{\tau_u}\right)\Bigg]d\tau_1,$$

$$f_{ut}=\frac{1}{\tau_u\langle u_i'(\mathbf{x},\ \tau)t'(\mathbf{x},\ \tau)\rangle}\int\limits_0^\infty\langle u_i'(\mathbf{R}_p(\tau_1),\ \tau_1)t'(\mathbf{x},\ \tau)\rangle\times$$
$$\times\exp\left(-\frac{\tau-\tau_1}{\tau_u}\right)d\tau_1,$$

$$f_t=\frac{1}{\tau_t\langle t'(\mathbf{x},\ \tau)t'(\mathbf{x},\ \tau)\rangle}\int\limits_0^\infty\langle t'(\mathbf{x},\ \tau)t'(\mathbf{R}_p(\tau_1),\ \tau_1)\rangle\times$$
$$\times\exp\left(-\frac{\tau-\tau_1}{\tau_t}\right)d\tau_1.$$

As can be seen from these relations, for calculating the coefficients of particle entrainment by turbulent pulsations of the carrying flow f and g one needs to know two-time correlation functions of velocity and temperature pulsations of the gas along the particle trajectories. The Lagrangian integral scale of turbulence T_L, characterizing the attenuation of energy-intensive pulsations of the carrying flow in time, may be taken as the time of particle interaction with the turbulent pulsations of the gas in a first approximation (with no substantial averaged interphase slip). In this case the coefficients of entrainment turn out to be functions of parameters of particle sluggishness Ω and Ω_t.

Substituting (11) into (3), we obtain the closed equation for the PDF of particles in a turbulent flow

$$\begin{aligned}&\frac{\partial P}{\partial\tau}+v_k\frac{\partial P}{\partial x_k}+\frac{\partial}{\partial v_k}\left(\frac{U_k-v_k}{\tau_u}+F_k\right)P+\frac{\partial}{\partial\vartheta}\left(\frac{T-\vartheta}{\tau_t}+Q\right)P=\\&=\frac{f_u}{\tau_u}\langle u_i'u_k'\rangle\frac{\partial^2P}{\partial v_i\partial v_k}+\left(\frac{f_{tu}}{\tau_u}+\frac{f_{ut}}{\tau_t}\right)\langle u_k't'\rangle\frac{\partial^2P}{\partial v_k\partial\vartheta}+\frac{f_t}{\tau_t}\times\\&\times\langle t'^2\rangle\frac{\partial^2P}{\partial\vartheta^2}+g_u\langle u_i'u_k'\rangle\left(\frac{\partial^2P}{\partial x_i\partial v_k}+\frac{\partial V_n}{\partial x_k}\frac{\partial^2P}{\partial v_i\partial v_n}+\frac{\partial\Theta}{\partial x_k}\frac{\partial^2P}{\partial v_i\partial\vartheta}\right)+\\&+\frac{\tau_u}{\tau_t}g_{ut}\langle u_i't'\rangle\left(\frac{\partial^2P}{\partial x_i\partial\vartheta}+\frac{\partial V_n}{\partial x_k}\frac{\partial^2P}{\partial v_n\partial\vartheta}+\frac{\partial\Theta}{\partial x_k}\frac{\partial^2P}{\partial\vartheta^2}\right).\end{aligned}\tag{12}$$

Ignoring the last two terms on the right side, (12) becomes a Fokker–Planck-type equation in the theory of Brownian particles. The role of the last two terms is especially essential for small particles ($g_u = g_{ut} \to 1/\Omega_u$ as $\Omega_u \to 0$). Equation (12) differs from that for the PDF obtained in [16] by the presence in the last two terms of the components containing spatial derivatives of the averaged velocity and temperature of particles. Due to the presence of these components the relation (9) becomes integrodifferential.

2. From (12) the equations for the moments of velocity and temperature of the dispersed phase can be obtained. The equations for the averaged concentration, velocity and temperature of particles coincide with those obtained in [16] and have the form:

$$\frac{\partial\Phi}{\partial\tau}+\frac{\partial\Phi V_k}{\partial x_k}=0,\tag{13}$$

$$\frac{\partial V_i}{\partial \tau} + V_k \frac{\partial V_i}{\partial x_k} = -\frac{\partial \langle v_i' v_k' \rangle}{\partial x_k} + \frac{U_i - V_i}{\tau_u} + F_i - \frac{D_{ik}}{\tau_u} \frac{\partial \ln \Phi}{\partial x_k}, \tag{14}$$

$$\frac{\partial \Theta}{\partial \tau} + V_k \frac{\partial \Theta}{\partial x_k} = -\frac{\partial \langle v_k' \vartheta' \rangle}{\partial x_k} + \frac{T - \Theta}{\tau_t} + Q - \frac{D_k^t}{\tau_t} \frac{\partial \ln \Phi}{\partial x_k}. \tag{15}$$

Here

$$\Phi = \iint P dv d\vartheta, \; V_i = \frac{1}{\Phi} \iint v_i P dv d\vartheta, \quad \Theta = \frac{1}{\Phi} \iint \vartheta P dv d\vartheta,$$

$$\langle v_i' v_j' \rangle = \frac{1}{\Phi} \iint v_i' v_k' P dv d\vartheta, \quad \langle v_k' \vartheta' \rangle = \frac{1}{\Phi} \iint v_k' \vartheta' P dv d\vartheta,$$

$$D_{ij} = \tau_u \left(\langle v_i' v_k' \rangle + g_u \langle u_i' u_k' \rangle \right), \; D_k^t = \tau_t \langle v_k' \vartheta' \rangle + \tau_u g_{ut} \langle u_k' t' \rangle .$$

It is pertinent to note that, owing to the presence of the second components (containing the coefficients g), the turbulent diffusion tensor D_{ik} and the diffusive heat transfer vector D_k^t with decreasing particle size tend to finite values rather than to zero ($D_{ik} \to T_L \langle u_i' u_k' \rangle$ and $D_k^t \to T_L \langle u_k' t' \rangle$ as $\Omega \to 0$), thus providing the limiting transition to a description of diffusion of an inertialess impurity in a turbulent flow.

The equations for the second moments of velocity and temperature pulsations have the form

$$\frac{\partial \langle v_i' v_j' \rangle}{\partial \tau} + V_k \frac{\partial \langle v_i' v_j' \rangle}{\partial x_k} + \frac{1}{\Phi} \frac{\partial \Phi \langle v_i' v_j' v_k' \rangle}{\partial x_k} = P_{ij} + \tag{16}$$

$$+ \frac{2}{\tau_u} \left(f_u \langle u_i' u_j' \rangle - \langle v_i' v_j' \rangle \right), \; P_{ij} = -\left(\langle v_i' v_k' \rangle \frac{\partial V_j}{\partial x_k} + \langle v_j' v_k' \rangle \frac{\partial V_i}{\partial x_j} \right),$$

$$\frac{\partial \langle v_i' \vartheta' \rangle}{\partial \tau} + V_k \frac{\partial \langle v_i' \vartheta' \rangle}{\partial x_k} + \frac{1}{\Phi} \frac{\partial \Phi \langle v_i' v_k' \vartheta' \rangle}{\partial x_k} = -\langle v_i' v_k' \rangle \frac{\partial \Theta}{\partial x_k} - \tag{17}$$

$$- \langle v_k' \vartheta' \rangle \frac{\partial V_i}{\partial x_k} + \left(\frac{f_{tu}}{\tau_u} + \frac{f_{ut}}{\tau_t} \right) \langle u_i' t' \rangle -$$

$$- \left(\frac{1}{\tau_u} + \frac{1}{\tau_t} \right) \langle v_i' \vartheta' \rangle ,$$

$$\frac{\partial \langle \vartheta'^2 \rangle}{\partial \tau} + V_k \frac{\partial \langle \vartheta'^2 \rangle}{\partial x_k} + \frac{1}{\Phi} \frac{\partial \Phi \langle v_k' \vartheta'^2 \rangle}{\partial x_k} = \tag{18}$$

$$= -2 \langle v_k' v' \rangle \frac{\partial \Theta}{\partial x_k} + \frac{2}{\tau_t} \left(f_t \langle t'^2 \rangle - \langle \vartheta'^2 \rangle \right).$$

The terms related to the last two ones in the equation for the PDF (12) are missing in Eqs. (16)-(18). From these equations there follow correct limiting relations as relaxation times of particles tend to zero - the second moments of velocity and temperature pulsations for inertialess particles are equal to the corresponding moments of pulsations of the carrying phase ($f \to 1$ when $\Omega \to 0$). Unlike (16)-(18), the equations for the second moments, given in [15, 16], contain additional terms which are due to the first components in the last two terms of Eq. (12) and describe additional generation of pulsations in a nonuniform flow, these additional terms persisting as the relaxation time of particles tends to zero and thus resulting in the emergence of special features in the behavior of the second moments of particle velocity and temperature pulsations with $\Omega \to 0$. The introduction into the equation for the PDF of terms directly related to the nonuniformity of the averaged velocity and temperature fields of the dispersed phase made it possible to eliminate redundant components in the equations for the second moments of pulsations and to provide a correct limiting transition for small particles ($\Omega \downarrow 0$).

Equations for the third moments of pulsations of particle velocity and temperature are written using the assumption of the near normal pulsation distribution law. In this case the fourth moments of pulsations can be approximately expressed as the sum of the products of the second moments, and the equations for the third moments will take the form:

$$\frac{\partial \langle v_i' v_j' v_k' \rangle}{\partial \tau} + V_n \frac{\partial \langle v_i' v_j' v_k' \rangle}{\partial x_n} + \langle v_i' v_j' v_n' \rangle \frac{\partial V_k}{\partial x_n} + \langle v_i' v_k' v_n' \rangle \frac{\partial V_j}{\partial x_n} + \langle v_j' v_k' v_n' \rangle \frac{\partial V_i}{\partial x_n} + \frac{D_{in}}{\tau_u} \frac{\partial \langle v_j' v_k' \rangle}{\partial x_n} + \frac{D_{jn}}{\tau_u} \frac{\partial \langle v_i' v_k' \rangle}{\partial x_n} + \frac{D_{kn}}{\tau_u} \frac{\partial \langle v_i' v_j' \rangle}{\partial x_n} + \frac{3}{\tau_u} \langle v_i' v_j' v_k' \rangle = 0, \tag{19}$$

$$\frac{\partial \langle v_i' v_j' \vartheta' \rangle}{\partial \tau} + V_k \frac{\partial \langle v_i' v_j' \vartheta' \rangle}{\partial x_k} + \langle v_i' v_k' \vartheta' \rangle \frac{\partial V_j}{\partial x_k} + \langle v_j' v_k' \vartheta' \rangle \frac{\partial V_i}{\partial x_k} + \langle v_i' v_j' v_k' \rangle \frac{\partial \Theta}{\partial x_k} + \frac{D_{ik}}{\tau_u} \frac{\partial \langle v_j' \vartheta' \rangle}{\partial x_k} + \frac{D_{jk}}{\tau_u} \frac{\partial \langle v_i' \vartheta' \rangle}{\partial x_k} + \frac{D_k^t}{\tau_t} \frac{\partial \langle v_i' v_j' \rangle}{\partial x_k} + \left(\frac{2}{\tau_u} + \frac{1}{\tau_t} \right) \langle v_i' v_j' \vartheta' \rangle = 0, \tag{20}$$

$$\frac{\partial \langle v_i' \vartheta'^2 \rangle}{\partial \tau} + V_k \frac{\partial \langle v_i' \vartheta'^2 \rangle}{\partial x_k} + \langle v_k' \vartheta'^2 \rangle \frac{\partial V_i}{\partial x_k} + 2 \langle v_i' v_k' \vartheta' \rangle \frac{\partial \Theta}{\partial x_k} + \frac{D_{ik}}{\tau_u} \frac{\partial \langle \vartheta'^2 \rangle}{\partial x_k} + 2 \frac{D_k^t}{\tau_t} \frac{\partial \langle v_i' \vartheta' \rangle}{\partial x_k} + \left(\frac{1}{\tau_u} + \frac{2}{\tau_t} \right) \langle v_i' \vartheta'^2 \rangle = 0. \tag{21}$$

3. With the known characteristics of the carrying turbulent flow the system (13)-(21) gives a closed description of momentum and heat transfer in the dispersed phase at the level of equations for the third moments. To simplify the calculation procedure and to realize the description of hydrodynamics and heat transfer at the level of equations for the second moments, assuming Eqs. (19)-(21) contain small terms, determining a time variation, convective transfer, and generation of the third moments of pulsations at the expense of the gradients of the averaged velocity and temperature, we obtain the following algebraic relations for the third moments:

$$\langle v_i' v_j' v_k' \rangle = -\frac{1}{3} \left(D_{in} \frac{\partial \langle v_j' v_k' \rangle}{\partial x_n} + D_{jn} \frac{\partial \langle v_i' v_k' \rangle}{\partial x_n} + D_{kn} \frac{\partial \langle v_i' v_j' \rangle}{\partial x_n} \right), \tag{22}$$

$$\langle v_i' v_j' \vartheta' \rangle = -\frac{1}{\tau_u + 2\tau_t} \left(\tau_t D_{ik} \frac{\partial \langle v_j' \vartheta' \rangle}{\partial x_k} + \tau_t D_{jk} \frac{\partial \langle v_i' \vartheta' \rangle}{\partial x_k} + \tau_u D_k^t \frac{\partial \langle v_i' v_j' \rangle}{\partial x_k} \right), \tag{23}$$

$$\langle v_i' \vartheta'^2 \rangle = -\frac{1}{2\tau_u + \tau_t} \left(\tau_t D_{ik} \frac{\partial \langle \vartheta'^2 \rangle}{\partial x_k} + 2\tau_u D_k^t \frac{\partial \langle v_i' \vartheta' \rangle}{\partial x_k} \right). \tag{24}$$

It is evident from Eqs. (19)-(21) that the exactness of the relations (22)-(24) increases with decreasing particle relaxation times. As the relaxation times tend to zero, expression (22) becomes the relation proposed in [18, 19] for determining the third moments of velocity pulsations in a single-phase medium.

The system of Eqs. (13)-(18), in view of the relations (22)-(24), gives a closed description of momentum and heat transfer in the dispersed phase at the level of equations for the second moments. A further simplification of the calculation procedure can be associated with the replacement of the system of equations for the second moments of velocity pulsations (16) by one differential equation for the dispersed phase turbulent energy

$$\frac{\partial k_p}{\partial \tau} + V_k \frac{\partial k_p}{\partial x_k} + \frac{1}{\Phi} \frac{\partial}{\partial x_k} \left(\Phi \frac{\langle v_i' v_i' v_k' \rangle}{2} \right) = \frac{P_{kk}}{2} + \frac{2}{\tau_u} (f_u k - k_p), \tag{25}$$

where in accordance with (22)

$$\frac{\langle v_k' v_k' v_i' \rangle}{2} = -\frac{1}{3} \left(D_{kn} \frac{\partial \langle v_i' v_k' \rangle}{\partial x_n} + D_{in} \frac{\partial k_p}{\partial x_n} \right). \tag{26}$$

For going from the system of differential equations (16) to that of algebraic equations use is made of the Rody transform, which has become wide-spread for single-phase turbulent flows [20]:

$$\begin{gathered} \frac{\partial \langle v_i' v_j' \rangle}{\partial \tau} + V_k \frac{\partial \langle v_i' v_j' \rangle}{\partial x_k} + \frac{1}{\Phi} \frac{\partial \Phi \langle v_i' v_j' v_k' \rangle}{\partial x_k} = \\ = \frac{\langle v_i' v_j' \rangle}{k_p} \left[\frac{\partial k_p}{\partial \tau} + V_k \frac{\partial k_p}{\partial x_k} + \frac{1}{\Phi} \frac{\partial}{\partial x_k} \left(\Phi \frac{\langle v_n' v_n' v_k' \rangle}{2} \right) \right]. \end{gathered} \tag{27}$$

From (16), (25), and (27) we obtain the following system of algebraic equations for determining the second moments of velocity pulsations:

$$\langle v_i' v_j' \rangle = \frac{2}{3} k_p \delta_{ij} + k_p \frac{f_u \left(\langle u_i' u_j' \rangle - \frac{2}{3} k \delta_{ij} \right) + \frac{\tau_u}{2} \left(P_{ij} - \frac{P_{kk}}{3} \delta_{ij} \right)}{f_u + \tau_u P_{kk}/4}. \tag{28}$$

Equations. (13), (14), and (25), with allowance for (26) and (28), give the description of hydrodynamics of the dispersed phase based on a differential equation of turbulent energy balance and on the system of equations for the second moments of velocity pulsations. To obtain an explicit expression for $\langle v_i' v_j' \rangle$, additional simplifications can be made, for which purpose we take the "isotropic" representation for the generation tensor:

$$P_{ij} = -\frac{2}{3} k_p \left(\frac{\partial V_i}{\partial x_j} + \frac{\partial V_j}{\partial x_i} \right)$$

and, ignoring the left side of Eq. (25), assume

$$k_p = f_u k + \tau_u P_{kk}/4.$$

Allowing for the taken relations, from (28) it follows

$$\begin{gathered} \langle v_i' v_j' \rangle = \frac{2}{3} k_p \delta_{ij} + f_u \left(\langle u_i' u_j' \rangle - \frac{2}{3} k \delta_{ij} \right) - \\ - \frac{\tau_u k_p}{3} \left(\frac{\partial V_i}{\partial x_j} + \frac{\partial V_j}{\partial x_i} - \frac{2}{3} \frac{\partial V_k}{\partial x_k} \delta_{ij} \right). \end{gathered} \tag{29}$$

The second and the third terms in (29), which determine anisotropic components of the tensor $\langle v_i' v_j' \rangle$, are caused by the direct involvement of particles in pulsation motion of the gas and by the generation of turbulent pulsations from the averaged motion of the dispersed phase; the contribution of these terms is determining for small and large particles, respectively. Thus, assuming in the approximation of the second term, that for small particles the averaged velocities of the dispersed and carrying phases are equal $V_i \approx U_i$, expression (29) can be presented as the Boussinesq relation

$$\langle v_i' v_j' \rangle = \frac{2}{3} k_p \delta_{ij} - \nu_p \left(\frac{\partial V_i}{\partial x_j} + \frac{\partial V_j}{\partial x_i} - \frac{2}{3} \frac{\partial V_k}{\partial x_k} \delta_{ij} \right), \tag{30}$$

where the turbulent viscosity factor is equal to

$$\nu_p = f_u \nu_T + \tau_u k_p / 3. \tag{31}$$

With allowance for the "isotropic" representation $\langle v_i' v_j' \rangle = 2k_p \delta_{ij}/3$, the diffusive term (26) in the equation of turbulent energy balance (25) is also simplified:

$$\frac{\langle v_i' v_i' v_k' \rangle}{2} = -\frac{10}{27} \tau_u (k_p + g_u k) \frac{\partial k_p}{\partial x_k}.$$

The simplest calculation procedure is obtained if in relation (29) or (31), for determining the turbulent energy of particles, the equilibrium (valid, as evident from (25), for a steady-state uniform flow or for small particles) relation $k_p = fk$ is taken. In this case (13) and (14), in view of (29) or (30), give a description of momentum transfer in the dispersed phase at the level of equations for the first moments.

The system of differential equations for a turbulent heat flow (17) may also be reduced to a system of algebraic equations if unsteady, convective, and diffusive terms are disregarded:

$$\begin{aligned} \langle v_i' \vartheta' \rangle = \frac{\tau_u f_{ut} + \tau_t f_{tu}}{\tau_u + \tau_t} \langle u_i' t' \rangle - \frac{\tau_u \tau_t}{\tau_u + \tau_t} \Big(\langle v_i' v_k' \rangle \frac{\partial \Theta}{\partial x_k} + \\ + \langle v_k' \vartheta' \rangle \frac{\partial V_i}{\partial x_k} \Big). \end{aligned} \tag{32}$$

The first and the second terms in (32) are due to the particle interaction with turbulent pulsations of the gas and to the generation of pulsations from the averaged motion of the dispersed phase; the role of these terms is determining for small and large particles, respectively. With the aim of obtaining an explicit expression for $\langle v_i' \vartheta' \rangle$, the last component related to the averaged velocity gradient is to be neglected in (32). In this case, with allowance for the "isotropic" presentation $\langle v_i' v_j' \rangle = 2k_p \delta_{ij}/3$ and the equality of the averaged temperatures of small particles and gas $\Theta \approx T$, expression (32) can be presented as the Fourier law:

$$\begin{aligned} \langle v_i' \vartheta' \rangle = - \left[\frac{(\tau_u f_{ut} + \tau_t f_{tu}) \nu_T}{(\tau_u + \tau_t) \mathrm{Pr}_T} + \frac{2\tau_u \tau_t}{3(\tau_u + \tau_t)} k_p \right] \frac{\partial \Theta}{\partial x_i} = \\ = - \frac{\nu_p}{\mathrm{Pr}_p} \frac{\partial \Theta}{\partial x_i}, \end{aligned} \tag{33}$$

where the turbulent Prandtl number for the dispersed phase is equal to:

$$\mathrm{Pr}_p = \frac{(\tau_u + \tau_t)(f_u \nu_T + \tau_u k_p/3)}{(\tau_u f_{ut} + \tau_t f_{tu}) \nu_T / \mathrm{Pr}_T + 2\tau_u \tau_t k_p / 3}. \tag{34}$$

According to (34), there exist the limiting relations

$$\mathrm{Pr}_p \to \mathrm{Pr}_T \text{ when } \Omega \to 0, \quad \mathrm{Pr}_p \to \frac{\tau_u + \tau_t}{2\tau_t} \text{ when } \Omega \to \infty.$$

The algebraic expression for the intensity of particle temperature pulsations can be obtained when the left side in Eq. (18) is disregarded

$$\langle \vartheta'^2 \rangle = f_t \langle t'^2 \rangle - \tau_t \langle v_k' \vartheta' \rangle \frac{\partial \Theta}{\partial x_k}, \tag{35}$$

or in view of relation (33)

$$\langle \vartheta'^2 \rangle = f_t \langle t'^2 \rangle + \tau_t \frac{\nu_p}{\mathrm{Pr}_p} \frac{\partial \Theta}{\partial x_k} \frac{\partial \Theta}{\partial x_k}. \tag{36}$$

The first term in expressions (35) and (36) is determining for small particles, the role of the second one grows with increasing particle size.

Thus, the presented models enable one at a different complexity level to describe the hydrodynamics and heat transfer of the dispersed phase in turbulent flows.

NOTATION

τ, time; R_p, V_p, ϑ_p, coordinate, velocity, and temperature of particle; v_i, V_i, v_i', ϑ, Θ, ϑ', actual, averaged, and pulsation components of velocity and temperature of dispersed phase; u_i, U_i, u_i', t, T, t′, actual, averaged, and pulsation components of velocity and temperature of carrying flow; F_i, external force; Q, density of internal heat sources in particle; τ_u, τ_i, times of dynamic and thermal relaxation of particle; $\Omega_u = \tau_u/T_L$, $\Omega_t = \tau_t/T_L$, parameters of dynamic and thermal sluggishness of particles; Φ, volume concentration of dispersed phase; $k = \langle u_i' u_j' \rangle/2$, turbulent energy of gas; ν_T, Pr_T, turbulent viscosity factor and turbulent Prandtl numbers for gaseous phase; $\delta(x)$, Dirac delta function; $\eta(x)$, Heaviside unit function.

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